Programming Assignment

Due: Tuesday, May 30, 2017, by 11:59p

You can use any programming language you prefer (MATLAB, Python, C/C++, or Julia, for example). Write down your code as clearly as possible and add suitable comments. For the submission, please follow the instruction below.

- Summarize the answers concisely in a document of any extension (e.g. hw5-ans.doc, hw5-ans.pdf). If you cannot get an answer because your code does not run, please comment your progress in the answer file.
- Please zip your code and the answer file in one file with the exact name "hw5-Last name-First name.zip".
- Submit your zip file to `ece154ucsd@gmail.com` with the exact subject `ECE 154C (HW5)`.

1. In this homework, we examine the performance of (7,4) Hamming code compared to other (7,4) codes over BSC(p), the binary symmetric channel with bit error probability p. Throughout this problem, we consider the systematic generator matrix G of the following form: $G = [I_k|A]$ where I_k is the $k \times k$ identity matrix and A is a $k \times (n-k)$ matrix. For example, for the (7,4) Hamming code, we consider

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}.$$

In this case, recall that the parity check matrix H is obtained easily by $H = [A^T|I_{n-k}]$.

(a) Recall that the minimum distance decoding is maximum likelihood decoding for BSC(p), where $p < 1/2$. Write a minimum distance decoder function `mindistdec(y,C)`. This function takes a received bits `y` and a codebook `C` as inputs. (To implement this, you may need to write a function for generating the codebook for a given generator matrix G.)

(b) Write a syndrome decoder function `syndromedec(y,H)`, which takes a received bits `y` and a parity check matrix `H` as inputs.

(c) For the $(7,4)$ Hamming code, verify that any one bit error in code bits can be corrected by both `mindistdec` and `syndromedec`. Hence, for simplicity, we now use `mindistdec` as our decoder for the rest of this problem.

(d) (NOT a programming) For the $(7,4)$ Hamming code, find the formula of the probability of decoding to a wrong codeword when the code is used for BSC(p).

(e) We want to run a simulation that verifies the result found in Problem (c). For $p = 0.05 : 0.05 : 0.5$, do the following steps 1,000 times to get a good estimate of the decoded word error rate (WER).

   i. Generate 4 random binary digits.
   ii. Encode them by calculating the 3 check digits, or using generator matrix G.
   iii. Put the codeword through BSC(p).
   iv. Decode the received 7-bit vector to a 4-bit word.
   v. Compare the decoded word with the transmitted word and count word errors to find word error rate (WER). Also count the errors in the information positions of the code words to obtain an estimate of the decoded bit error probability (BER).

   Plot your simulation results p vs. BER(p) and p vs. WER(p), along with the analytical expression found in part (d). Is the bit error probability larger or smaller than the word error probability?

(f) Now we consider another systematic generator matrix

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}.$$

   Repeat the same experiment as in (e), and compare the results with BER and WER of the (7,4) Hamming code.

(g) Finally we examine the performance of a randomly generated $(7, 4)$ systematic generator matrices. Generate a random 4×3 binary matrix A and use $G = [I_4, A]$ as our generator matrix. For each G, repeat the same experiment in (e), and take averages of BERs and WERs over the random matrices. Compare the results with BER and WER of the (7,4) Hamming code. Is there any p where a randomly generated G performs better than the Hamming (7,4) code on average?

(Note: you do not have to compute the theoretical expression of WER for parts (f) and (g).)