Programming Assignment

Due: Thursday, May 18, 2017, by 11:59p

You can use any programming language you prefer (MATLAB, Python, C/C++, or Julia, for example). Write down your code as clearly as possible and add suitable comments. For the submission, please follow the instruction below.

- Summarize the answers concisely in a document of any extension (e.g. hw4-ans.doc, hw4-ans.pdf). If you cannot get an answer because your code does not run, please comment your progress in the answer file.
- Please zip your code and the answer file in one file with the exact name "hw4-Last name-First name.zip".
- Submit your zip file to `ece154ucsd@gmail.com` with the exact subject `ECE 154C (HW4)`.

1. Write a program for a function `computeCapacity(channel)` that takes the channel matrix `channel` as an input and outputs the capacity of the channel and the capacity achieving input distribution as a vector. For example, for a channel

$$p(y|x) = \begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.2 & 0.5 & 0.3 \\ 0.3 & 0.2 & 0.5 \end{bmatrix},$$

your function should output the capacity $4/5 + 1/2 * \log 5 - 3/10 * \log 3 \approx 1.4855$ bits per transmission, and the capacity achieving input distribution $p^*(x) = [1/3, 1/3, 1/3]$.

   (a) Find the capacity and the capacity achieving input distribution for a channel

$$p(y|x) = \begin{bmatrix} 0.5 & 0.3 & 0.2 & 0 \\ 0.3 & 0.5 & 0 & 0.2 \\ 0.5 & 0 & 0.3 & 0.2 \\ 0.2 & 0 & 0.5 & 0.3 \end{bmatrix}$$

   (b) Find the capacity and the capacity achieving input distribution for a channel

$$p(y|x) = \begin{bmatrix} 0.4 & 0.4 & 0.2 \\ 0.3 & 0.3 & 0.4 \\ 0.2 & 0.5 & 0.3 \\ 0.2 & 0.2 & 0.6 \end{bmatrix}$$

   (c) In this problem, we will consider a random channel. Suppose the input and output alphabets are $\{1, 2, 3, 4, 5, 6\}$. We define a random channel as for each j

$$Y|\{X = j\} = \begin{cases} j & \text{with probability (w.p.) } 1/2, \\ Z_j & \text{w.p. } 1/2, \end{cases}$$

where Z_j is drawn uniformly from $\{1,2,3,4,5,6\}\backslash\{j\}$, and Z_j's are independent. For example,

$$\frac{1}{2}\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

is one possible realization of the channel.

Generate 100 random channels as above and find a capacity for each channel using your function `computeCapacity`. Plot the empirical CDF of capacity. Find the upper bound and the lower bound based on your experiment, and comment on the results.